Photon counts statistics of squeezed and multi-mode thermal states of light on multiplexed on-off detectors

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Photon number resolving detectors can be highly useful for studying the statistics of multi-photon quantum states of light. In this work we study the counts statistics of different states of light measured on multiplexed on-off detectors. We put special emphasis on artificial nonclassical features of the statistics obtained. We show new ways to derive analytical formulas for counts statistics and their moments. Using our approach we are the first to derive statistics moments for multi-mode thermal states measured on multiplexed on-off detectors. We use them to determine empirical Mandel parameters and recently proposed subbinomial parameters suitable for tests of nonclassicality of the measured states. Additionally, we investigate subpoissonian and superbunching properties of the two-mode squeezed state measured on a pair of multiplexed detectors and we present results of the Fano factor and second-order correlation function for these states.

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1. Introduction

Multi-photon states of light are highly applicable in precise quantum metrology [1–3]. Among these states, the N00N states [4, 5] and the squeezed states [6] are of the particular importance. The latter type of states have recently been successfully applied, for instance in increasing the precision of gravitational interferometer [7] or in low-noise quantum imaging [8].

Recent advances in technology allow for measurement of photon statistics of light with photon number resolving (PNR) detectors. They have lived to see many implementations among which the most popular are multiplexed on-off detectors based on the photon chopping concept [9]. They have been manufactured as fiber loop detectors [10–12], multi-pixel photon counters (MPPC) [13] or as single photon-sensitive cameras [14, 15]. Other types of detectors also in use include calorimetric transition-edge sensors [16] and hybrid photo-detectors [17–19]. The mutltiplexed on-off detectors have clear advantages such as a fast response and a relatively easy construction, since many of them are based on fast avalanche photodiodes.

Up till now there have been a number of successful experiments using PNR detectors, thus developing further the knowledge on quantum states of light and its sources [17–21]. Many other experiments may be enhanced by the use of PNR detectors, for example, the observation of macro-micro entanglement [22] or quantum imaging [8].

Here in this paper we will focus on the counting properties of the multiplexed on-off detectors, which are the type of PNR detectors most often used. These detectors alter the counts statistics as compared to the photon statistics of light illuminating a detector [9, 23–25]. A modification in counts statistics often leads to seemingly nonclassical properties of measured light [21, 26]; therefore, an appropriate interpretation of the counts statistics is indispensable.

In particular when it comes to an experiment, adequate criteria of nonclassicality based on empirical counts statistics have to be applied [27–29]. Criteria of nonclassicality for counts typically allow us to determine qualitatively whether one is dealing with a quantum state or a classical one. In order to better differentiate between these states, the counts statistics models are indispensable. In many cases the measured state can be identified only based on the analysis of the mean and the variance of counts.

In this paper we put a special emphasis on the multimode thermal states of light for which we are the first to derive the analytical model of counts statistics. Currently there is a great interest in the community to observe multimode light, not only generated in the spontaneous parametric down-conversion, but also in atomic systems including room-temperature alkali metals vapors [30–33]. These experiments could be naturally reimplemented into PNR regime with the use of single-

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photon sensitive cameras as in [34].

Furthermore, in this article we present the numerical model for two-mode squeezed states observed on two identical PNR detectors. We present the results for nonclassical measures such as the Fano factor and the second order correlation function, commonly used in experiments.

2. Detector counting properties

Multiplexed detectors consist of a finite number of N Geiger-type on-off detectors. Number of detectors N is the parameter which determines the counting statistics of the detector. N is also the maximum number of counts that can be measured.

The detector operation can be described by giving the conditional probability of obtaining k counts provided that the detector was illuminated by n photons [9]:

$$p_N(k|n) = \frac{1}{N^n}\binom{N}{k} k! S(n,k), \tag{1}$$

where $S(n,k) = \frac{1}{k!}\sum_{i=0}^{k}(-1)^i \binom{k}{i}(k-i)^n$ is the Stirling number of the second kind [35]. For instance, Eq. (1) yields $p_N(0|n) = 0$ for all $n \geq 1$ and $p(1|n) = \frac{1}{N^{n-1}}$.

Typically it is enough to know the conditional probabilities $p_N(k|n)$ for a finite value of n since the detector saturates for large n as we illustrated in Fig. 1 (a-b).

Due to finite quantum efficiency η of the detector the model Eq. 1 has to include photon losses $p_\eta(n|m) = \binom{m}{n}\eta^n(1-\eta)^{m-n}$.

This can also be done analytically for specific n and k, for instance $p_N(0|n) = (1-\eta)^n$ or

$$p_N(1|n) = N(1-\eta)^n\left(\left(\frac{\eta - \eta N + N}{N - \eta N}\right)^n - 1\right). \tag{2}$$

We have gathered a whole set of conditional probabilities in Fig. 1 (c-d).

Here in this paper we derive an analytical and numerical model for an idealized detector subject to certain assumptions. We shall assume that the component detectors are very similar, thus principally they will have the same quantum efficiency η. Moreover, the dark counts and the cross talk between the component on-off detectors will be excluded from our model.

If one cannot apply these assumptions to the used detector, another, alternative approach should be used such as Bayessian approach to treat the cross-talk [13, 36, 37] Including all imperfections in a general form to the theoretical model would make it very complicated and of little use. Instead, one can find the values of $p_N(k|n)$ through detector tomography [24, 26, 38].

3. Counts statistics on a single detector

In the experiment with the PNR detector, we have access to the history of counts collected by the detector, which yields empirical probability distributions for counts statistics c_k. It can be expressed as $c_k = \sum_{n=k}^{\infty} p_N(k|n) f_n$, where f_n is the photon number distribution in the measured state.

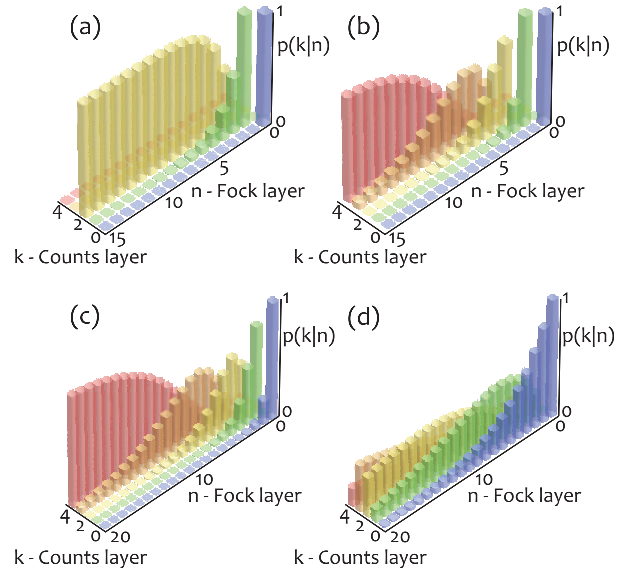

Figure 1. Values of conditional probabilities $p_N(k|n)$ of getting k counts when a detector is illuminated by exactly n photons, for a detector consisting of N multiplexed component on-off detectors with quantum efficiency η. (a) $N = 2$, $\eta = 1$ (b) (a) $N = 4$, $\eta = 1$ (c) $N = 4$, $\eta = 0.8$ (d) $N = 4$, $\eta = 0.2$.

Detector losses can be taken into account both by an appropriate modification of the conditional probabilities of the detector $p_N(k|n)$ and, as described earlier, in the photons statistics.

In the following discussion we shall assume the statistics f_n after losses and the detector will be described by no-loss conditional probabilities as in Eq. (1).

In the experiment we often confine ourselves to tracking only the mean and the variance of the counts statistics. Determination of these values is often sufficient to identify the measured state. Now, we shall derive the moments of probability distributions for the counts.

In general we can find average values of polynomial functions $\chi(k)$ of counts:

$$\langle \chi(k) \rangle = \sum_{k=0}^{N} \chi(k)\binom{N}{k} k! \sum_{n=k}^{\infty} \frac{1}{N^n} S(n,k) f_n \tag{3}$$

To evaluate Eq. (3) we can use two useful properties of Stirling number of the second kind [35]:

$$\sum_{n=k}^{\infty} S(n,k)\frac{x^n}{n!} = \frac{1}{k!}(e^x - 1)^k \tag{4}$$

$$\sum_{n=k}^{\infty} S(n,k) x^n = \frac{(-1)^k}{(1-\frac{1}{x})_k} \tag{5}$$

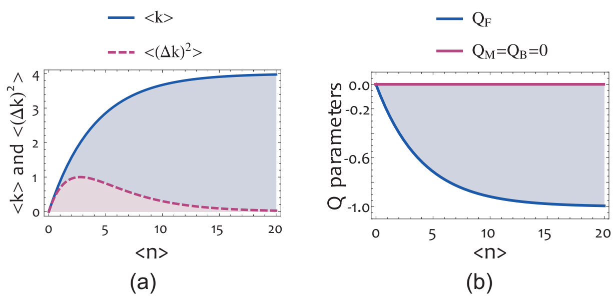

Figure 2. (a) Mean and variance of counts for coherent state on a detector with $N = 4$. (b) Measured Q_F parameter is artificially nonclassical contrary to $Q_B = Q_M = 0$.

where $(\cdot)_n$ denotes the Pochhammer symbol: $(y)_n \equiv \Gamma(y+n)/\Gamma(y)$.

These two properties facilitate the determination of means and variances for states of light frequently used in experiments. For instance, using the property Eq. (4) one can find the mean $\langle k\rangle$ and the variance $\langle(\Delta k)^2\rangle$ of counts for the coherent state $f_n^{\mathrm{coh}} = \frac{\langle n\rangle^n}{n!}e^{-\langle n\rangle}$:

$$\langle k\rangle = N(1 - e^{-\langle n\rangle/N}) \tag{6}$$

$$\langle(\Delta k)^2\rangle = N(1 - e^{-\langle n\rangle/N})e^{-\langle n\rangle/N} \tag{7}$$

Here we see that the detector reduces both the mean and the variance of counts as compared with the values for the photon statistics Fig. 2 (a).

This reduction leads to the seemingly nonclassical properties of the measured light. One of nonclassicality criteria for single-mode light is the negativity of the Mandel parameter $Q_M = \langle(\Delta n)^2\rangle/\langle n\rangle - 1$. The parameter can also be evaluated for counts statistics $Q_F = \langle(\Delta k)^2\rangle/\langle k\rangle - 1$, here for coherent states being always negative $Q_F = e^{-\langle n\rangle/N} - 1 < 0$ (Fig. 2 (b)).

A recently proposed modified criterion of nonclassicality is based on the subbinomial parameter [21, 27]:

$$Q_B = N\frac{\langle(\Delta k)^2\rangle}{\langle k\rangle(N - \langle k\rangle)} - 1,$$

negative only for nonclassical states and for coherent states reconstructing the true value of Q_M Mandel parameter (Fig. 2 (b)).

4. Multi-mode thermal states of light

Now we proceed to calculations for single and multi-mode thermal states of light which can viewed as a single subsystem of the squeezed states of light.

At first, let us consider the following photons statistics:

$$f_n = ab^n, \tag{8}$$

which can be readily associated with the single mode thermal state of the mean $\langle n\rangle$ for which $a = (1+\langle n\rangle)^{-1}$ and $b = \langle n\rangle/(1+\langle n\rangle)$.

Then, using the property Eq. (5), we find the average of any function of counts for given statistics f_n:

$$\langle\chi(k)\rangle_f = a\sum_{k=0}^{N}\chi(k)\binom{N}{k}k!\frac{(-1)^k}{(1-N/b)_k}. \tag{9}$$

We focus particularly on the first and second moment of the counts statistics:

$$\langle k\rangle_f = \frac{abN}{(b-1)(b(N-1)-N)} \tag{10}$$

$$\langle k^2\rangle_f = \frac{ab(b+1)N^2}{(b-1)(b(N-2)-N)(b+N-bN)}. \tag{11}$$

We are also able to find moments of counts distribution for another type of statistics yielded from Eq. (8), in particular:

$$g_n = a\frac{(n+m)!}{n!}b^{n+m}, \tag{12}$$

which can be expressed as:

$$g_n = a\partial_{b,m}b^{n+m} = \partial_{b,m}b^m f_n. \tag{13}$$

Note that g_n can be readily associated with the multi-mode thermal state with an average number of photons $\langle n\rangle$ and the number of modes $\mathcal{M}$ is described by the statistics g_n Eq. (12) for $a = \frac{1}{\Gamma(\mathcal{M})}\left(\frac{\mathcal{M}}{\langle n\rangle+\mathcal{M}}\right)^{\mathcal{M}}$, $b = \langle n\rangle/(\mathcal{M}+\langle n\rangle)$ and $m = \mathcal{M}-1$ [39].

To calculate the moments for given statistics g_n we simply do the following:

$$\langle\chi(k)\rangle_g = \partial_{b,m}b^m\langle\chi(k)\rangle_f \tag{14}$$

In this way, we can find the mean and the variance for a single-mode thermal state, where$a = (1+\langle n\rangle)^{-1}$ and $b = \langle n\rangle/(1+\langle n\rangle)$, similarly as in [27]:

$$\langle k\rangle_{\mathrm{Th},1} = \frac{\langle n\rangle N}{\langle n\rangle + N}$$

$$\langle(\Delta k)^2\rangle_{\mathrm{Th},1} = \frac{\langle n\rangle N^2(\langle n\rangle N + \langle n\rangle + N)}{(\langle n\rangle + N)^2(2\langle n\rangle + N)}$$

Equations (14), (10) and (11) and substitutions for a and b yield analytical expressions for the mean $\langle k\rangle_{\mathrm{Th},\mathcal{M}}$ and the variance $\langle(\Delta k)^2\rangle_{\mathrm{Th},\mathcal{M}}$ for each $\mathcal{M}$ and N. For

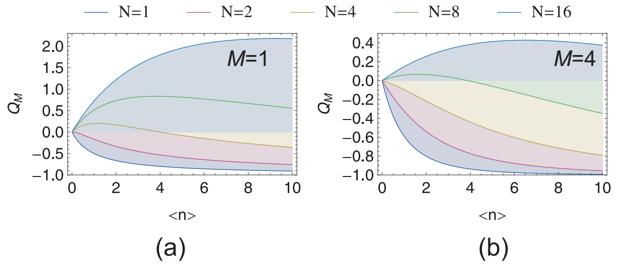

Figure 3. Theoretical values for empirical Q_F parameters for (a) a single-mode thermal state, (b) multi-mode thermal state $\mathcal{M}=4$ plotted versus mean number of photons $\langle n\rangle$ for different N. Depending on the parameters of the state and the detector counts statistics may seem classical or artificially nonclassical.

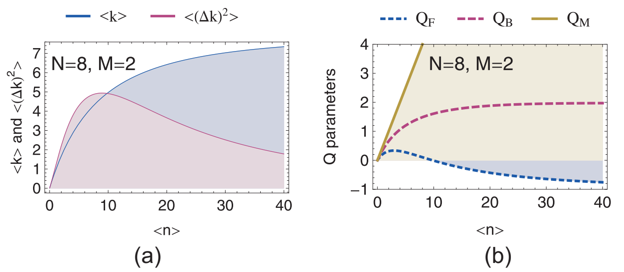

Figure 4. (a) Mean and variance of counts, (b) input Q_M, measured Q_F and binomial Q_B parameters versus mean number of photons in two-mode ($\mathcal{M}=2$) thermal state on a detector with $N=8$.

example, we give analytical expressions for $\langle k\rangle_{\mathrm{Th},\mathcal{M}}$ for two-mode $\mathcal{M}=2$ thermal state:

$$\langle k\rangle_{\mathrm{Th},\mathcal{M}=2}=\frac{\langle n\rangle N(\langle n\rangle+4N)}{(\langle n\rangle+2N)^2}.$$

Further the second moment $\langle k^2\rangle_{\mathrm{Th},\mathcal{M}=2}$ can be expressed analytically:

$$\langle k^2\rangle_{\mathrm{Th},\mathcal{M}=2}= =\frac{\langle n\rangle N^2\left(\langle n\rangle^3+6\langle n\rangle^2N+3\langle n\rangle N(2N+1)+4N^2\right)}{(\langle n\rangle+N)^2(\langle n\rangle+2N)^2}$$

Analytical formulas for a higher number of modes can be readily found using any symbolic computation software and instead of presenting directly the formulas we gather the results for higher number of modes on plots.

Having found means and variances parameters, we can construct Mandel parameters for counts and compare them with theoretical values $Q_M=\langle n\rangle/\mathcal{M}$. In Fig. 3 we show the empirical Q_F parameter for detectors of different N for single- and multi-mode thermal state ($\mathcal{M}=4$). Depending on the mean number of input photons $\langle n\rangle$ and N, the measured Q_F parameters appear to be nonclassical is certain regimes.

5. Counts properties of squeezed states

The above results can be measured in a single subsystem for single- and multi-mode squeezed states using a single detector. If we have two detectors, we can calculate the joint counts statistics c_{k_1,k_2} related to the input statistics f_{n_1,n_2} by relation:

$$c_{k_1,k_2}=\sum_{n_1=k_1}^{\infty}\sum_{n_2=k_2}^{\infty}p_N(k_1|n_1)p_N(k_2|n_2)f_{n_1,n_2}$$

Calculations of the joint count statistics allow us to detect subpoissonian correlations between the two subsystems. A good measure to quantify these correlations is the Fano factor [18] which, evaluated for counts, can be expressed as:

$$R=\frac{\langle(\Delta(k_1-k_2))^2\rangle}{\langle k_1\rangle+\langle k_2\rangle},$$

which is $R\geq 1$ for all classical states.

Another commonly used parameter to characterize in experiments the two modes states of light is the second order correlation function, which evaluated on counts statistics can be expressed as:

$$g^{(2)}=\frac{\langle k_1k_2\rangle}{\langle k_1\rangle\langle k_2\rangle}.$$

It can be used to test if the investigated light demonstrate the superbunching properties i.e. for $g^{(2)}>2$, which is characteristic for squeezed vacuum states [40] but in principle may be also obtained using interference of thermal states [41].

Now we shall focus on properties of joint counts statistics of the two-mode squeezed state of light:

$$|\psi\rangle=\sqrt{1-|\zeta|^2}\sum_{n=0}^{\infty}\zeta^n|nn\rangle,$$

where ζ is the squeezing parameter. The Fano factor for such a state without losses equals $R=0$, whereas the second orderd correlation function becomes always over two: $g^{(2)}=1+1/|\zeta|^2\geq 2$.

In Fig. 5 we show the effect of a finite number N of component detectors and influence of losses on the joint counts statistics c_{k_1,k_2}, for $\zeta=0.8$.

Both the finite number of detectors and the losses have influence on reducing the correlation between counts measured on two detectors. To better understand this mechanism, we found the Fano factors for different N, η and ζ. Fig. 6 (a) presents pure influence of a finite N, without the losses.

For small squeezing parameters ζ high number of component detectors N ensures a smaller Fano factor, whereas for high ζ the effect is opposite. This is the effect of artificial increasing of correlations due to the detector saturation. The saturation effects are even more

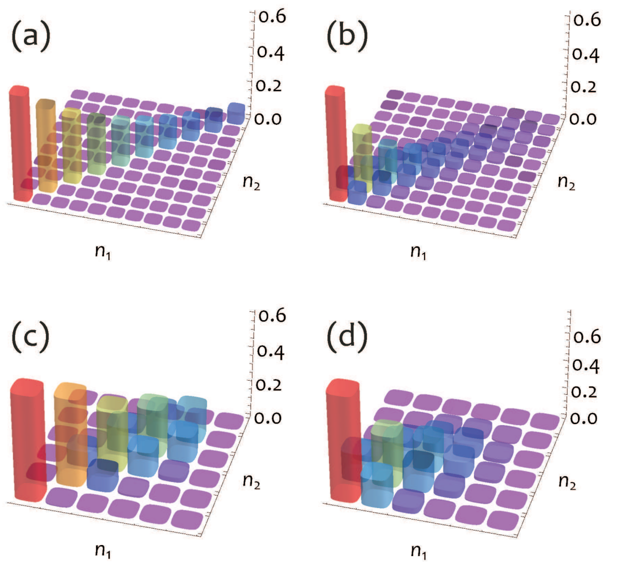

Figure 5. Two mode squeezed state with $\zeta = 0.8$ measured by two multiplexed detectors. (a) $N = \infty$, $\eta = 1$. (b) $N = \infty$, $\eta = 0.8$. (a) $N = 4$, $\eta = 1$. (a) $N = 4$, $\eta = 0.5$.

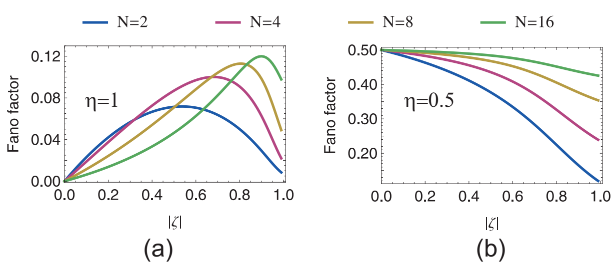

Figure 6. Fano factors versus squeezing parameter ζ for different numbers of N detectors with (a) $\eta = 1$ (b) $\eta = 0.5$. A lower number of detectors N can artificially increase the correlation and consequently decrease the Fano factor.

significant for the detector with finite quantum efficiency (Fig. 6 (b)), where $\eta = 0.5$. Here only a smaller number of detectors N has an effect on reducing the Fano factor for eachζ.

In all these cases, the calculated counts statistics always remain nonclassical. In order to perform a reliable test of nonclassicality with no *a priori* knowledge of the state of light, one can apply the recently proposed criteria [29].

It is also instructive to view how the second correlation function $g^{(2)}$ is modified due to the limitations introduced by a finite number of component on-off detectors N. It can be seen in Fig. 7 that the low number of component on-off detectors N decreases $g^{(2)}$ by no more than 1. This means that limit between superbunched states and the bunched states ($g^{(2)} = 2$) will be exceeded for a sufficiently low N for squeezed states of the squeezing parameter higher than $|\zeta|^2 > 1/2$ as it is exemplified in Fig. 7 (b).

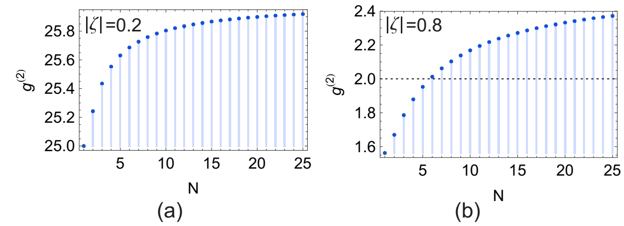

Figure 7. Second order correlation function $g^{(2)}$ versus different numbers N of detectors for $\eta = 1$ for $|\zeta| = 0.2$ (a) and for $|\zeta| = 0.8$ (b) for two-mode squeezed states. Finite number N of component on-off detectors decreases $g^{(2)}$ which for two-mode squeezed state always is higher than two, $g^{(2)} > 2$. $g^{(2)}$ is decreased by no more than one. For $|\zeta|^2 > 1/2$ and sufficiently low N, the observed counts statistics does not preserve its initial superbunching properties as in (b).

6. Summary

In this paper, we investigate the statistics for counts measured by a detector illuminated by squeezed or thermal states of light, the latter being a traced subsystem of the squeezed state.

In particular, we provide a universal manner of deriving statistics moments which we have used to determine analytical formulas for multi-mode thermal states. In this work we put special emphasis on the issues related to artificial nonclassicality of measured statistics in the context of Mandel and subbinomial parameters.

On the other hand, we show the influence of increasing and decreasing subpoissonian counts correlations for two-mode squeezed states of light measured on two multiplexed detectors. We show that the Fano factor always remains nonclassical. Moreover we provide the results for the second-order correlation function $g^{(2)}$ which could drop below two in certain cases.

The results of the work can be used to identify the states of light measured by multiplexed on-off detectors, in particular to determine the number of modes in thermal states. The results can significantly contribute to designing experiments using multiplexed on-off detectors. They will play an increasingly important role in the future since the state of the art, photon number resolving detectors are becoming a very efficient tool for studying the multimode, multi-photon quantum states of light.

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